2019 Semester 1: **Mid-Term Equation Sheet Hand in at Exam End**

NAME: ____________________ ID #: ______________

$$m_r = \frac{1}{N}\sum_{i=1}^{N}[x_i - \bar{x}]^r,\ s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}[x_i - \bar{x}]^2}$$

$$skew_{KS} = \frac{N^2}{(N-1)(N-2)}\frac{m_3}{s^3}, \qquad \sqrt{\frac{(N-1)(N-2)}{6N}} \cdot skew_{KS} \overset{A}{\sim} N(0,1),$$

$$kurt_{KS} = \left[\frac{N^2}{(N-1)(N-2)(N-3)}\left(\frac{(N+1)m_4 - 3(N-1)m_2^2}{s^4}\right)\right],$$

$$\sqrt{\frac{(N-1)(N-2)(N-3)}{24N(N+1)}} \cdot kurt_{KS} \overset{A}{\sim} N(0,1),$$

$$\hat{\rho}_{xy} = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2 \cdot \sum_{i=1}^{N}(y_i - \bar{y})^2}}.$$

$$\frac{\bar{X} - \mu}{\sqrt{\frac{s^2}{N}}} \sim t_{N-1}, \qquad \frac{\hat{\rho} - 0}{\sqrt{1 - \hat{\rho}^2}\frac{1}{\sqrt{N-2}}} \sim t_{N-2}.$$

$$z = \frac{1}{2}\ln\left(\frac{1+\hat{\rho}}{1-\hat{\rho}}\right), \quad \zeta_{H_0} = \frac{1}{2}\ln\left(\frac{1+\rho_{H_0}}{1-\rho_{H_0}}\right), \quad \frac{z - \zeta_{H_0}}{\sqrt{\frac{1}{N-1} + \frac{2}{(N-1)^2}}} \ \sim \ N(0,1)$$

$$t_k = \frac{N(0,1)}{\sqrt{\frac{\chi_k^2}{k}}}, \qquad \chi_\nu^2 = Z_1^2 + Z_2^2 + \ldots + Z_\nu^2, \qquad F_{\nu,\eta} = \frac{\chi_\nu^2/\nu}{\chi_\eta^2/\eta}$$

$$\begin{aligned}
c(t) &= S(t)e^{-\rho(T-t)}N(d_1) - e^{-r(T-t)}XN(d_2),\\
p(t) &= e^{-r(T-t)}XN(-d_2) - S(t)e^{-\rho(T-t)}N(-d_1),\\
d_1 &= \frac{\ln\left(\frac{S(t)}{X}\right) + (r - \rho + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}, \text{ and}\\
d_2 &= d_1 - \sigma\sqrt{T-t}.
\end{aligned}$$

$$p = c = \frac{S\sigma\sqrt{T-t}}{\sqrt{2\pi}} \approx 0.4S\sigma\sqrt{T-t}$$

$$R_t = \frac{P_t}{P_{t-1}} - 1, \quad r_t \equiv ln(1 + R_t) = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

$$\vec{R} = \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_N \end{pmatrix}, \ \vec{h} = \begin{pmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{pmatrix}, \ V = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \ldots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \ldots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \ldots & \sigma_{NN} \end{pmatrix},$$

$$R_P = \vec{h}'\vec{R},\ E(R_P) = \vec{h}'E(\vec{R}), \text{ and } \sigma_P^2 = \vec{h}'V\vec{h}.$$

$$R_n = R_F + \beta_n(R_B - R_F) + \rho_n,$$
$$R_P = R_F + \beta_P(R_B - R_F) + \rho_P.$$

$$\begin{aligned} R_P - R_B &= (R_P - R_F) - (R_B - R_F) \\ &= [\beta_P(R_B - R_F) + \rho_P] - (R_B - R_F) \\ &= (\beta_P - 1)(R_B - R_F) + \rho_P \\ &= \beta_P^{(A)} \cdot (R_B - R_F) + \rho_P \\ \psi_P &= \sqrt{V(R_P - R_B)} \\ &= \sqrt{(1-\beta_P)^2 \cdot \sigma_B^2 + \omega_P^2}. \end{aligned}$$

$$\begin{aligned} \rho_P &= (R_P - R_F) - \beta_P \cdot (R_B - R_F) \\ \omega_P &= \sqrt{V(\rho_P)} = \sqrt{\sigma_P^2 - \beta_P^2\sigma_B^2}. \end{aligned}$$

$$\begin{aligned} \kappa_P &= \overbrace{\kappa_B}^{\text{benchmark return}} + \overbrace{(\beta_P - 1)(\mu_B + \delta_B)}^{\text{benchmark timing}} + \overbrace{\alpha_P}^{\text{stock selection}} \\ \sigma_P^2 &= \underbrace{\sigma_B^2}_{\text{benchmark risk}} + \underbrace{(\beta_P^2 - 1)\sigma_B^2}_{\text{benchmark timing}} + \underbrace{\omega_P^2}_{\text{stock selection}} \end{aligned}$$

$$RAA = \alpha_P - \lambda_R\omega_P^2,\ \ RTAA = \alpha_P - \lambda_R\omega_P^2 - TC$$

$$\vec{h}_a\text{: } \vec{\iota}\,'\vec{h}_a = 1,\ \mu_a = \vec{h}_a'\vec{\mu},\ \sigma_a^2 = \vec{h}_a'V\vec{h}_a,\ \sigma_{ab} = \vec{h}_a'V\vec{h}_b.$$

$$\vec{h}_P = \frac{1}{D}\left\{[C(V^{-1}\vec{\mu}) - A(V^{-1}\vec{\iota})]\mu_P + [B(V^{-1}\vec{\iota}) - A(V^{-1}\vec{\mu})]\right\}$$

$$A = \vec{\iota}\,'V^{-1}\vec{\mu},\ \ B = \vec{\mu}\,'V^{-1}\vec{\mu},\ \ C = \vec{\iota}\,'V^{-1}\vec{\iota}, \ \ \text{and } D = BC - A^2 \text{ (all }_{(1\times 1)}).$$

$$\vec{h}_P = \underbrace{\frac{\mu_P - R_F}{(\vec{\mu} - R_F\vec{\iota})'V^{-1}(\vec{\mu} - R_F\vec{\iota})}V^{-1}(\vec{\mu} - R_F\vec{\iota})}_{\text{column vector of weight in risky assets for given } \mu_P},\ \vec{h}_T = \tfrac{V^{-1}(\vec{\mu} - R_F\vec{\iota})}{\vec{\iota}\,'V^{-1}(\vec{\mu} - R_F\vec{\iota})}.$$

$$r_i(t) = \underbrace{\vec{B}_i'}_{(1\times K)} \cdot \underbrace{\vec{R}(t)}_{(K\times 1)} + u_i, \qquad V = \underbrace{\mathcal{B}}_{(N\times K)} \cdot \underbrace{F}_{(K\times K)} \underbrace{\mathcal{B}'}_{(K\times N)} + \underbrace{\Delta}_{(N\times N)},$$

$$r_n = \mu\tau + \sigma\sqrt{\tau}\ z_n$$

$$\hat{\mu} = \frac{1}{N}\sum_{n=1}^{N} r_n \Big/ \tau \ = \ \sum_{n=1}^{N} r_n\ , \quad \hat{\sigma}^2 = \frac{1}{N}\sum_{n=1}^{N} r_n^2 \Big/ \tau \ = \ \sum_{n=1}^{N} r_n^2$$

$$\begin{aligned} E(\hat{\mu}) &= \mu \\ V(\hat{\mu}) &= \sigma^2 \\ E(\hat{\sigma}^2) &= \sigma^2 + \frac{\mu^2}{N} \quad \xrightarrow{N} \sigma^2 \\ V(\hat{\sigma}^2) &= \frac{2\sigma^4}{N} + \frac{4\mu^2\sigma^2}{N} \quad \xrightarrow{N} 0 \end{aligned}$$

$$\begin{aligned}[r_{P10,t} - r_{F,t}] &= \alpha + \beta_{P10}[r_{B,t} - r_{F,t}] + \epsilon_t \\ \Rightarrow r^{(A)}_{P10,t} &= \alpha + \beta^{(A)}_{P10}[r_{B,t} - r_{F,t}] + \epsilon_t \\ \Rightarrow \bar{r}_{P10} &= \bar{r}_B + \hat{\alpha} + \hat{\beta}^{(A)}_{P10}[\bar{r}_B - \bar{r}_F]\end{aligned}$$

$$PVA = \frac{C}{R}\left[1 - \frac{1}{(1+R)^N}\right]$$

$$PVGA = \frac{C}{R-g}\left[1 - \left(\frac{1+g}{1+R}\right)^N\right]$$

$$IR = IC \times \sqrt{BR}$$

$$P_0 = \frac{\text{EPS}}{R} + \text{PVGO and } \frac{P_0}{\text{EPS}} = \frac{1}{R}\left(\frac{1}{1 - \frac{\text{PVGO}}{P}}\right)$$

$$P^*_t = B_t + \sum_{i=1}^{\infty} \frac{\text{EVA}_{t+i}}{(1+r_e)^i},$$

$$B_{t+1} = B_t + NI_{t+1} - D_{t+1},\ EVA_{t+i} = NI_{t+1} - r_e B_{t+i-1}$$

$$\frac{P^*_t}{B_t} = 1 + \sum_{i=1}^{\infty}\left[\left(\frac{ROE_{t+i} - r_e}{(1+r_e)^i}\right) \cdot \left(\frac{B_{t+i-1}}{B_t}\right)\right]$$

$$PV = \frac{E(\tilde{c}) - cov(\tilde{c}, R_M) \cdot (E(R_M) - R_F)/\sigma^2_M}{1 + R_F} \text{ and } PV = \frac{E^*(\tilde{c})}{1+R_F},$$

$$\begin{aligned} r_{it} &= \alpha_i + \beta_i r_{bt} + u_{it} \\ u_{it}|\mathcal{F}_{i\ t-1} &\sim \mathcal{N}(0, h_{it}) \\ h_{it} &= \gamma_{0i} + \gamma_{1i} u^2_{i\ t-1} + \gamma_{2i} h_{i\ t-1}. \end{aligned}$$

$$(ABM) \quad P_t = \mu + P_{t-1} + \epsilon_t, \quad \epsilon_t \text{ IID } \mathcal{N}(0, \sigma^2)$$

$$(GBM) \quad \ln P_t = \mu + \ln P_{t-1} + \epsilon_t, \quad \epsilon_t \text{ IID } \mathcal{N}(0, \sigma^2)$$

$$[E(P_{t+1}|P_t, P_{t-1}, \ldots) = P_t] \Rightarrow [E(P_{t+1} - P_t|P_t, P_{t-1}, \ldots) = 0]$$

$$\text{Dimson} \quad R_{j,t} = \alpha + \beta_{-1} R_{M,t-1} + \beta_0 R_{M,t} + \beta_{+1} R_{M,t+1} + \epsilon_{j,t}$$

$$\beta_{Dimson} = \hat{\beta}_{-1} + \hat{\beta}_0 + \hat{\beta}_{+1}$$

Scholes-Williams for each of $\Delta t = -1, 0, 1$, $R_{j,t} = \alpha + \beta_{\Delta t} R_{M,t+\Delta t} + \epsilon_{j,t}$, where

$$\beta_{SW} = \frac{\hat{\beta}_{-1} + \hat{\beta}_0 + \hat{\beta}_{+1}}{1 + 2\hat{\rho}(1)}, \text{ and } \hat{\rho}(1) \text{ is auto-corr of index}$$

$$R_{SMB,t} = \left(\frac{R_{S/L,t} + R_{S/M,t} + R_{S/H,t}}{3}\right) - \left(\frac{R_{B/L,t} + R_{B/M,t} + R_{B/H,t}}{3}\right).$$

$$R_{HML,t} = \left(\frac{R_{S/H,t} + R_{B/H,t}}{2}\right) - \left(\frac{R_{S/L,t} + R_{B/L,t}}{2}\right),$$

$$E(R_i) - R_f = b_i[E(R_M) - R_F] + s_i E(SMB) + h_i E(HML)$$

$$R_i - R_f = \alpha_i + b_i(R_M - R_F) + s_i SMB + h_i HML$$

$$R_i = E(R_i) + b_{i1}\delta_1 + \ldots + b_{iK}\delta_K + \epsilon_i, \quad \delta_1 = R_M - E(R_M)$$

$$E(R_i) = \lambda_0 + \lambda_1 b_{i1} + \ldots + \lambda_K b_{iK}, \quad \lambda_1 = E(R_M) - R_F$$

END